Recurrence Relation

A recurrence is an equation or inequality that describes a function in terms of its values on smaller inputs. To solve a Recurrence Relation means to obtain a function defined on the natural numbers that satisfy the recurrence.

**For Example**, the Worst Case Running Time T(n) of the MERGE SORT Procedures is described by the recurrence.

T (n) = θ (1) if n=1

 2T + θ (n) if n>1

There are four methods for solving Recurrence:

* Substitution Method
* Iteration Methods
* Recursion Tree Method
* Master Method

**Substitution Method:**

The Substitution Method Consists of two main steps:

1. Guess the Solution.
2. Use the mathematical induction to find the boundary condition and shows that the guess is correct.

**For Example1:**  Solve the equation by Substitution Method.

     T (n) = T + n

We have to show that it is asymptotically bound by O (log n).

**Solution:**

For T (n) = O (log n)

We have to show that for some constant c

1.   T (n) ≤c logn.

Put this in given Recurrence Equation.

     T (n) ≤c log+ 1

                     ≤c log+ 1 = c logn-clog2 2+1

                     ≤c logn for c≥1

Thus **T (n) =O logn**.

**Example2:**  Consider the Recurrence

T (n) = 2T+ n, n>1

Find an Asymptotic bound on T.

**Solution:**

We guess the solution is O (n (logn)). Thus, for constant 'c'.

 T (n) ≤c n logn

Put this in given Recurrence Equation.

Now,

 T (n) ≤2clog +n

     ≤cnlogn-cnlog2+n

     =cn logn-n (clog2-1)

     ≤cn logn for (c≥1)

Thus **T (n) = O (n logn)**.

**Iteration Methods:**

It means to expand the recurrence and express it as a summation of terms of n and initial condition.

**Example1:** Consider the Recurrence

1.   T (n) = 1 **if** n=1

2.         = 2T (n-1) **if** n>1

**Solution:**

T (n) = 2T (n-1)

     = 2[2T (n-2)] = 22T (n-2)

     = 4[2T (n-3)] = 23T (n-3)

     = 8[2T (n-4)] = 24T (n-4)  (Eq.1)

Repeat the procedure for i times

T (n) = 2i T (n-i)

Put n-i=1 or i= n-1 in   (Eq.1)

T (n) = 2n-1 T (1)

     = 2n-1 .1   {T (1) =1 .....given}

     = 2n-1

**Example2:** Consider the Recurrence

1.   T (n) = T (n-1) +1 and T (1) =  θ (1).

**Solution:**

 T (n) = T (n-1) +1

      = (T (n-2) +1) +1 = (T (n-3) +1) +1+1

      = T (n-4) +4 = T (n-5) +1+4

      = T (n-5) +5= T (n-k) + k

Where k = n-1

  T (n-k) = T (1) = θ (1)

  T (n) = θ (1) + (n-1) = 1+n-1=n= θ (n).

**RELEVANT READING MATERIAL AND REFERENCES:**

**Source Notes:**

1. <https://www.javatpoint.com/daa-recurrence-relation>

**Lecture Video:**

1. <https://youtu.be/4V30R3I1vLI>
2. <https://youtu.be/MhT7XmxhaCE>
3. <https://youtu.be/7lq-rBdM62o>

**Online Notes:**

1. <http://vssut.ac.in/lecture_notes/lecture1428551222.pdf>

**Text Book Reading:**

1. Cormen, Leiserson, Rivest, Stein, “*Introduction to Algorithms*”, Prentice Hall of India, 3rd edition 2012. problem, Graph coloring.

**In addition: PPT can be also be given.**